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A THEORETICAL STUDY OF ABLATION SHIELD REQUIREMENTS FOR MANNED REENTRY VEHICLES

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A THEORETICAL STUDY OF ABLATION SHIELD REQUIREMENTS

FOR MANNED REENTRY VEHICLES

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ABSTRACT

The problem of sublimation of material and accumulation of heat in an ablation shield is analyzed and applied to the reentry of manned vehicles into the earth's atmosphere. The parameters which control the amount of sublimation and the temperature distribution within the ablation shield are determined and presented in a manner useful for engineering calculation. It is shown that the total mass loss from the shield during reentry and the insulation requirements may be given very simply in terms of the maximum deceleration of the vehicle or the total reentry time.

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INTRODUCTION

The successful return of a vehicle through the earth's atmosphere depends largely on the provision that is made for reducing aerodynamic heat transfer to the structure of the vehicle. Analyses of the heating experienced during reentry have been made for both ballistic vehicles (ref. 1) and for manned vehicles (ref. 2).

For the purpose of the present report the types of reentry are categorized as follows:

(a) Lifting vehicles of constant lift-to-drag ratio which reenter the atmosphere at very small angles (so that skipping does not occur) and which experience maximum decelerations less than about 8g

(b) Nonlifting vehicles which reenter at small angles and which experience maximum decelerations between 8g and 14g

(c) Nonlifting vehicles which enter at larger angles and experience maximum decelerations greater than 14g (ballistic trajectory)

The vehicles considered in (a) and (b) are suitable for manned reentry whereas the decelerations associated with higher entry angles (type c) generally exceed human tolerances.

The heating experience of the manned vehicles also differs from that of the higher entry angle ballistic vehicle: for this latter vehicle the maximum heating rates are such that surface temperatures may exceed the melting temperatures of metals which have been considered in heat-sink type shields. For manned vehicles the flight duration is much longer and although the maximum heating rate is much lower the total heat input exceeds that of the ballistic vehicle and the use of a metal heat-sink shield becomes inefficient from a weight standpoint.

As an alternative to the heat sink, consideration has been given to the use of ablation materials; the term ablation applies when there is a removal of material (and an associated removal of heat) caused by aerodynamic heating, and therefore embraces melting, sublimation, melting and subsequent vaporization of the liquid film, burning or depolymerization.

Several approximate analyses have been made of the steady-state shielding effects which result from this removal of material; aerodynamic melting has been considered in references 3, 4, and 5; sublimation, in reference 6; simultaneous melting and vaporization, in references 7 and 8; and a general treatment of the boundary layer with mass addition has been given in reference 9. The problem of keeping the vehicle structure at a suitably low temperature cannot be answered by investigating a steady-state situation, however; consideration must be given to the problem of insulation and the conduction of heat to the structure is ~~an unsteady~~ a transient phenomenon.

The suitability of a heat shield (whether heat sink or ablation material) depends on the weight required to keep the structure below a given temperature and a simple quantity of merit such as the effective heat capacity or effective heat of ablation gives no indication of the severity of the insulation problem - the use of high-temperature ablation materials such as graphite, for example, which has high thermal conductivity, could lead to an intolerable heating condition although the effective heat capacity is higher than most materials. It is possible that the high-surface-temperature ablation materials experience less mass loss because of radiation cooling but this is not necessarily desirable since the high surface temperature makes the insulation problem more severe.

It is seen, therefore, that the problem of maintaining a cool vehicle structure is twofold; firstly, there must be adequate provision of material for ablation and, secondly, there must be sufficient insulation to prevent the structure becoming hot.

The effectiveness of an ablation material depends on its capability to dispose of heat by convection in the liquid melt, as latent heat, and by convection in gaseous form in the boundary layer; in this regard it has been shown (refs. 6 and 7) that, in general, an ablation shield is most effective when a large fraction of the mass loss undergoes vaporization. When sublimation takes place there is no liquid film, all the mass lost from the shield undergoes vaporization and subsequent convection in the high-temperature boundary layer thereby removing a large amount of heat. For this reason a material which undergoes sublimation rather than melting is generally the more efficient (apart from considerations of latent heat).

Naturally, the choice of ablation material will be dictated by the type of vehicle and its heating history during reentry. For a nonlifting vehicle whose dimensions are such that the heating rates experienced are too high to be balanced by radiative cooling it is advantageous to use a low ablation temperature material, thereby reducing the insulation problem. For lifting vehicles which experience lower heating rates over most of the vehicle surface the primary means of cooling would be radiative except at the leading edges where the limited use of a high-temperature ablation material would seem more appropriate - ablation would then take place only near the peak heating condition.

The materials here considered most suitable for the reentry of a manned nonlifting capsule are therefore those which undergo sublimation at a low temperature (say less than 1,500° R) and have low conductivity so that no further insulation is required. The absence of a liquid phase firstly insures that the material is removed in gaseous form, and therefore convects a large amount of heat from the shield, and secondly precludes the possibility of liquid film instability.

The purpose of the report is to develop an approximate method of solution for such a shield from which may be determined the total sublimation of material during reentry and the temperature distribution within the remaining shield; the ablation temperature is such that radiation may be neglected. The analysis is directed towards obtaining results useful for engineering purposes.

ANALYSIS

Motion and Heating During Reentry

An analysis of shallow reentry into the earth's atmosphere of both lifting and nonlifting vehicles has been made in reference 2; the results of interest for the present application are included here for completeness.

Assuming an exponential variation of density with altitude

$$\frac{\rho_\infty}{\rho^*} = e^{-\beta Y}$$

the equations of motion reduce to a single differential equation

$$\bar{u}Z'' - \left(Z' - \frac{Z}{\bar{u}}\right) = \frac{1 - \bar{u}^2}{\bar{u}Z} - (\beta r)^{1/2}\,\frac{L}{D} \tag{1}$$

where

$$Z = \frac{\rho^*}{2\frac{M}{C_DA}}\left(\frac{r}{\beta}\right)^{1/2}\bar{u}e^{-\beta Y}, \quad \bar{u} = \frac{u}{u_c}$$

The initial conditions appropriate to reentry from a circular orbit are

$$Z_i = 0, \quad Z_i' = (\beta r)^{1/2}\sin\varphi_i \tag{2}$$

at $\bar{u} = 1$.

All quantities of interest are expressed in terms of $\bar{u}$, Z, and $\frac{M}{C_DA}$ as follows (see appendix for list of symbols):

Horizontal deceleration: $-\frac{du}{dt} = g(\beta r)^{1/2}\bar{u}Z$ ft/sec^2 (3)

Resultant deceleration: $a = g(\beta r)^{1/2}\left[1 + \left(\frac{L}{D}\right)^2\right]^{1/2}\bar{u}Z$ ft/sec^2 (4)

Reynolds number: $\frac{Re_\infty}{l} = \frac{2(\beta g)^{1/2}}{\mu_\infty}\frac{M}{C_DA}Z$ (5)

Time: $t = (\beta g)^{-1/2}\bar{t}$ sec (6)

where

$$\bar{t} = \int_{\bar{u}}^{\bar{u}_i}(\bar{u}Z)^{-1}d\bar{u} \tag{7}$$

Stagnation enthalpy: $c_p\left(T_e - T_\infty\right) = \frac{1}{2}\bar{u}^2\frac{{u_c}^2}{gJ}$ Btu/lb (8)

Heat-transfer rate: $q_0 = 590k_I\left(\frac{M}{C_DAR}\right)^{1/2}\bar{q}_0$ Btu/sq ft sec (9)

where

$$\bar{q}_0 = \bar{u}^2(\bar{u}Z)^{1/2} \tag{10}$$

Heat absorbed: $Q_0 = 15{,}900k_{II}\left(\frac{M}{C_DAR}\right)^{1/2}\bar{Q}_0$ Btu/sq ft (11)

where

$$\bar{Q}_0 = \int_{\bar{u}}^{\bar{u}_i}\bar{u}^2(\bar{u}Z)^{-1/2}d\bar{u} \tag{12}$$

The foregoing relations (eqs. (3) to (12)) are used in the determination of the mass required for sublimation and the accumulation of heat within the solid shield.

General Equations for Sublimation and Heat Accumulation

Before the vehicle reenters the atmosphere the ablation shield is assumed to have uniform temperature T_∞; in the early part of reentry the shield is heated until the surface temperature reaches the ablation temperature T_a. During this preablation heating period the problem of conduction of heat through the shield can be treated without difficulty since the heating rate is known (eq. (10)). When sublimation of material occurs, however, the conduction of heat within the material depends on the rate of mass loss from the surface, the problem becomes nonlinear and the exact solution involves lengthy numerical procedures (see, for example, ref. 10).

It is not the purpose of this report to obtain exact solutions of the nonlinear equations; rather, approximate results are obtained which show all the important parameters that enter into the problem and give estimates of the material required for sublimation and for absorbing the heat conducted to the interior. (Fig. 1 is a diagram of the heat shield under consideration.)

It is now assumed that the ablation shield is sufficiently thick that the "infinitely thick slab" approximation may be used and an energy balance is written as follows:

$$\underbrace{Q(t)}_{\substack{\text{Net heat}\\ \text{input at}\\ \text{surface}}} = \underbrace{\left[c_b(T_a - T_\infty) + L\right]m}_{\substack{\text{Heat absorbed by}\\ \text{sublimated material}}} + \underbrace{\rho_b c_b \int_{-\infty}^{0} (T - T_\infty)\,dz}_{\substack{\text{Heat accumulated by}\\ \text{remaining material}}} \qquad (13)$$

When an integral thickness θ defined as

$$\theta = \int_{-\infty}^{0} \frac{T - T_\infty}{T_s - T_\infty}\, dz \tag{14}$$

is introduced, equation (13) may be written in differential form as

$$q(t) = \left[c_b(T_a - T_\infty) + L\right]\frac{dm}{dt} + \rho_b c_b \frac{d}{dt}\left[(T_s - T_\infty)\theta\right] \tag{15}$$

and an additional equation (a boundary condition at the surface of sublimation) is written

$$q(t) = L\frac{dm}{dt} + \left(k_b \frac{\partial T}{\partial z}\right)_{z=0} \tag{16}$$

Net heat-transfer rate to surface	Rate of heat absorbed in phase change	Heat-transfer rate to interior

It is important to note that in equations (15) and (16) the heat-transfer rate $q(t)$ is that which the shield actually experiences and is itself a function of the rate of sublimation; throughout this report the quasi-steady relation for the reduction in heat-transfer rate, due to the introduction of mass into the boundary layer, is used (see ref. 6)

$$q_0(t) - q(t) = \alpha\tilde{c}_p(T_e - T_a) \tag{17}$$

where $q_0(t)$ is the heat-transfer rate experienced by a nonablating body at the surface temperature T_a.

In equation (17), $\tilde{c}_p$ is the effective mean specific heat and $\alpha(T_e - T_a)$ the effective temperature rise of the mass convected in the boundary layer. The expressions for α and $\tilde{c}_p$ derived in reference 6 for a laminar boundary layer are

$$\alpha = 1 - \frac{1}{3} N_{Pr}^{-0.6} \tag{18}$$

and

$$\tilde{c}_p = c_{p,1}\tilde{W} + c_{p,2}(1 - \tilde{W}) \tag{19}$$

where $\tilde{W}$ is the effective concentration of the shield material in gaseous form in the boundary layer and is given as a function of N_{Sc} in reference 6.

The unknown heating rate $q(t)$ is eliminated from equations (15) and (16) by use of equation (17) to give

$$q_0 = \left[c_b(T_a - T_\infty) + L + \alpha\tilde{c}_p(T_e - T_a)\right]\frac{dm}{dt} + \rho_b c_b \frac{d}{dt}\left[(T_s - T_\infty)\theta\right] \tag{20}$$

and

$$q_0 = \left[L + \alpha\tilde{c}_p(T_e - T_a)\right]\frac{dm}{dt} + \left(k_b \frac{\partial T}{\partial z}\right)_{z=0} \tag{21}$$

Before attempting to take account of the conduction of heat within the shield it is useful to make a simple analysis of the sublimation which gives estimates of the total mass loss during reentry.

Sublimation of Material From the Shield

The purpose of an ablation shield is to reduce the heat-transfer rate at the surface from the aerodynamic rate q_0 to a value $\left(k_b \frac{\partial T}{\partial z}\right)_{z=0}$ by providing material which absorbs heat through latent heat of sublimation and convects heat in the gas boundary layer; this situation is reflected in equation (21). If this process is successful then $\left(k_b \frac{\partial T}{\partial z}\right)_{z=0} \ll q_0$ for most of the reentry and an upper limit to the rate of mass loss can be obtained by neglecting heat conduction in the solid.

From equation (21) with $\left(k_b \frac{\partial T}{\partial z}\right)_{z=0} = 0$

$$\frac{dm}{dt} = \frac{q_0}{L + \alpha\tilde{c}_p(T_e - T_a)} \tag{22}$$

Alternatively, by assuming that the rate of accumulation of heat, $\frac{d}{dt}\left[\rho_b c_b (T_s - T_\infty)\theta\right]$, is small compared with the rate of disposal of heat $\left[c_b(T_a - T_\infty) + L + \alpha\tilde{c}_p(T_e - T_a)\right]\frac{dm}{dt}$, equation (20) becomes

$$\frac{dm}{dt} = \frac{q_0}{c_p(T_a - T_\infty) + L + \alpha\tilde{c}_p(T_e - T_a)} \tag{23}$$

Equation (23) is the quasi-steady expression for the rate of mass loss but is not, in general, an upper limit.

With the aid of equations (3), (8), (9), and (10), equation (23) is written

$$\frac{dm}{dt} = \frac{1.18}{27} k_I \frac{\left(\frac{M}{C_D AR}\right)^{1/2}}{\alpha \frac{\tilde{c}_p}{c_{p,2}}}(\bar{u}Z)\left(-\frac{d\bar{m}}{d\bar{u}}\right) \text{ lb/sq ft sec} \tag{24}$$

where

$$-\frac{d\bar{m}}{d\bar{u}} = \left(1 - \frac{\lambda}{\bar{u}^2 + \lambda}\right)(\bar{u}Z)^{-1/2} \tag{25}$$

and

$$\left.\begin{aligned}
\lambda &= \frac{L}{\frac{1}{2}\frac{u_c^2}{gJ}\alpha\frac{\tilde{c}_p}{c_{p,2}}} && \text{(using eq. (22))} \\
&\text{or} \\
\lambda &= \frac{L + c_b(T_a - T_\infty)}{\frac{1}{2}\frac{u_c^2}{gJ}\alpha\frac{\tilde{c}_p}{c_{p,2}}} && \text{(using eq. (23))}
\end{aligned}\right\} \tag{26}$$

Equation (24) shows immediately the importance of the parameter

$$\frac{\left(\frac{M}{C_D AR}\right)^{1/2}}{\alpha\frac{\tilde{c}_p}{c_{p,2}}} \approx \frac{\text{Heating enthalpy}}{\text{Gas shielding enthalpy}}$$

which depends on the vehicle size and shape through $\frac{M}{C_D AR}$ and the properties of the gas boundary layer. It is seen that the mass loss can be reduced by designing the vehicle so that $\frac{M}{C_D AR}$ is small (low mass, high drag, blunt nose) and by choosing an ablation material having a high shielding coefficient, $\alpha \frac{\tilde{c}_p}{c_{p,2}}$.

A second parameter, the enthalpy ratio

$$\lambda = \frac{L}{\frac{1}{2}\frac{u_c^2}{gJ}\alpha\frac{\tilde{c}_p}{c_{p,2}}} = \frac{\text{Shielding due to latent heat}}{\text{Gas layer convective shielding}}$$

shows the effect of the latent heat L in reducing the mass loss; when $\alpha \frac{\tilde{c}_p}{c_{p,2}}$ is small the effect of latent heat becomes more important.

<u>Mass loss</u>.- An upper limit to the total mass loss is obtained by integration of equation (24)

$$m = 1.18k_{II}\frac{\left(\frac{M}{C_D AR}\right)^{1/2}}{\alpha\frac{\tilde{c}_p}{c_{p,2}}}\bar{m}\quad \text{lb/sq ft} \tag{27}$$

where

$$\bar{m} = \int_{\bar{u}_f}^{\bar{u}_a}\left(1 - \frac{\lambda}{\bar{u}^2 + \lambda}\right)(\bar{u}Z)^{-1/2}d\bar{u} \tag{28}$$

and $\bar{u}_a$, $\bar{u}_f$ are, respectively, the values of $\bar{u}$ when sublimation begins and ends.

Equation (27) with $k_{II} = 1$ gives the mass loss at the stagnation point and the factor $k_{II} = \frac{1}{S}\int\frac{q_0}{q_{s_{p,0}}}dS$ modifies this mass loss according to the variation of heating rate over the surface of the shield. The analysis of reference 2 does not apply at the condition $\bar{u} = 1$ and it is neccessary to assume a value $\bar{u} < 1$ as the upper limit of $\bar{u}$; the nominal value of $\bar{u} = 0.995$, used in reference 2, is also used here as the value at which sublimation begins. The lower limit depends on the ablation temperature of the material since ablation will cease before the stagnation temperature of the stream falls below the ablation temperature. The value $\bar{u}_f = 0.05$ (which corresponds to stream temperature of about 200° F) is used throughout this paper; this value is considered sufficiently low to include any material now under consideration.

Thus $\bar{m}$ can be written

$$\bar{m} = \bar{m}_{\lambda=0}(1 - \eta) \tag{29}$$

where

$$\bar{m}_{\lambda=0} = \int_{0.05}^{0.995} (\bar{u}Z)^{-1/2} d\bar{u} \tag{30}$$

and

$$\eta = \frac{\int_{0.05}^{0.995} \frac{\lambda}{\bar{u}^2 + \lambda} (uZ)^{-1/2} d\bar{u}}{\int_{0.05}^{0.995} (\bar{u}Z)^{-1/2} d\bar{u}} \tag{31}$$

Equation (30) shows that, even when $\lambda = 0$, (no latent heat) there is a limiting value of the total mass loss, whereas equation (31) gives $\eta(\lambda)$, the fractional decrease in mass loss due to latent heat.

<u>Effective heat capacity</u>.- When comparing an ablation shield with a solid "heat-sink" shield (for example, copper or beryllium) it is convenient to introduce an effective heat capacity defined here by the following ratio

$$H_{eff} = \frac{\text{Total heat which would be absorbed by a nonablating shield}}{\text{Total mass loss from ablation shield}}$$

Using equations (11) and (12) for Q_0 and (27), (29), and (30) for m, the effective heat capacity can be written

$$H_{eff} = 13{,}500\alpha \frac{\tilde{c}_p}{c_{p,2}} \bar{H}_{eff} \text{ Btu/lb} \tag{32}$$

where

$$\left.\begin{aligned} \bar{H}_{eff} &= \left(\bar{H}_{eff}\right)_{\lambda=0}(1 - \eta) \\ &\text{and} \\ \left(\bar{H}_{eff}\right)_{\lambda=0} &= \frac{\int_{0.05}^{0.995} \bar{u}^2(\bar{u}Z)^{-1/2}d\bar{u}}{\int_{0.05}^{0.995} (\bar{u}Z)^{-1/2}d\bar{u}} \end{aligned}\right\} \tag{33}$$

Equations (32) and (33) show that H_{eff} depends only on the properties of ablation material $\left(\text{through } \tilde{c}_p/c_{p,2} \text{ and } \lambda\right)$ and on the vehicle trajectory $\left(\text{since } \left(\bar{H}_{eff}\right)_{\lambda=0} \text{ is a function only of trajectory}\right)$.

Relation between mass loss, deceleration, and time of reentry.- From equations (24) and (25) $\frac{dm}{dt} \sim \frac{\bar{u}^2}{\bar{u}^2 + \lambda}(\bar{u}Z)^{1/2}$; thus for large λ, $\left(\frac{dm}{dt}\right)_{max}$ occurs at $\left[\bar{u}^2(\bar{u}Z)^{1/2}\right]_{max}$ (peak heating) whereas for $\lambda = 0$, $\left(\frac{dm}{dt}\right)_{max}$ occurs at $\left[(\bar{u}Z)^{1/2}\right]_{max}$ (peak horizontal deceleration). For any λ, then, the maximum sublimation rate occurs between peak heating and peak deceleration.

In general, the total mass loss will depend on the total time taken to complete reentry since $\bar{m}_{\lambda=0}$ is a function of the trajectory. It is of extreme interest, therefore, to determine how the mass loss may be reduced by allowing the vehicle to complete reentry in a short period of time but with the reservation that the maximum deceleration be kept to a tolerable level.

The relationship between the total mass loss, deceleration, and time of reentry becomes apparent when equations (3), (7), and (30) are recalled:

$$-\frac{1}{g}\frac{du}{dt} = (\beta r)^{1/2}\bar{u}Z, \quad \bar{t} = \int_{0.05}^{0.995} (\bar{u}Z)^{-1} d\bar{u}, \text{ and } \bar{m}_{\lambda=0} = \int_{0.05}^{0.995} (\bar{u}Z)^{-1/2} d\bar{u}$$

Firstly, since $(\bar{u}Z) \leqq (\bar{u}Z)_{max}$ it is seen that

$$\bar{m}_{\lambda=0} \geqq \left[\frac{\text{Max. horiz. decn., g's}}{(\beta r)^{1/2}}\right]^{-1/2}$$

secondly, since

$$\left[\int_0^1 (\bar{u}Z)^{-1/2} d\bar{u}\right]^2 \leqq \int_0^1 (\bar{u}Z)^{-1} d\bar{u} \qquad \text{(Schwartz integral inequality)}$$

then

$$\bar{m}_{\lambda=0} \leqq (\bar{t})^{1/2}$$

so that in general $\bar{m}_{\lambda=0}$ satisfies

$$\left[\frac{1}{30}\,\frac{\text{Max. horiz. decn., g's}}{2}\right]^{-1/2} \leqq \bar{m}_{\lambda=0} \leqq \left(\frac{t}{27}\right)^{1/2}$$

(inserting the numerical values of $(\beta r)^{1/2}$ and $(\beta g)^{1/2}$) a result independent of vehicle characteristics or trajectory. Using an average value of $\bar{u}Z$ equal to $\frac{1}{2}(\bar{u}Z)_{max}$ the following simple rule may be expected to hold

$$\bar{m}_{\lambda=0} \approx \left(\frac{1}{30}\,\frac{\text{Max. horiz. decn., g's}}{2}\right)^{-1/2} \tag{34}$$

that is to say,

A. The total mass loss varies inversely as the square root of the maximum horizontal deceleration.

Alternatively, comparing the integral expressions for t and $\bar{m}_{\lambda=0}$, the following relation is expected

$$\bar{m}_{\lambda=0} \sim \left(\frac{t}{27}\right)^{1/2}$$

that is,

B. The total mass loss varies as the square root of time for reentry.

The foregoing relations between mass loss, deceleration, and reentry time, provide a very simple estimate of the sublimation during reentry.

Application to particular vehicles.- The integrals required for $\bar{m}_{\lambda=0}$, $\left(\bar{H}_{eff}\right)_{\lambda=0}$, and η are evaluated for

(a) Lifting vehicles, $\frac{L}{D} > 1$, $\varphi_1 = 0$ for which $\bar{u}Z = \left(30 \frac{L}{D}\right)^{-1}(1 - \bar{u}^2)$.

(b) Nonlifting vehicle, $\frac{L}{D} = 0$, $-\varphi_1 < 5^o$, $\bar{u}Z$ given numerically in reference 2, and

(c) Nonlifting ballistic vehicle, $\frac{L}{D} = 0$, $-\varphi_1 < 6^o$ for which $\bar{u}Z = 30 \sin - \varphi_1\bar{u}^2(-\ln \bar{u})$.

The quantities $\bar{t}$ and maximum horizontal deceleration are also evaluated and a check made on the simple rules A and B; the results appear in the following table.

TABLE I

Vehicle	Entry angle $-\varphi_1$	Dimensionless mass loss, $\bar{m}_{\lambda=0}$	Rule A	Rule B	$\left(\bar{H}_{eff}\right)_{\lambda=0}$	η						
			$\left(\frac{1}{30}\ \frac{\text{Max. horiz. decn., g's}}{2}\right)^{-1/2}$	$0.84(\bar{t})^{1/2}$		$\lambda = 0$	$\lambda = 0.05$	$\lambda = 0.1$	$\lambda = 0.25$	$\lambda = 0.50$	$\lambda = 1.0$	$\lambda = \infty$
Lifting, $\frac{L}{D} > 1$	0	$1.418\left(30\ \frac{L}{D}\right)^{1/2}$	$1.414\left(30\ \frac{L}{D}\right)^{1/2}$	$1.44\left(30\ \frac{L}{D}\right)^{1/2}$	0.5	*0	*0.218	*0.302	*0.447	*0.577	*0.707	*1
Manned nonlifting, $\frac{L}{D} = 0$	0	3.06	2.70	3.69	0.500	0	0.223	0.305	0.450	0.580	0.707	1
	$\frac{1}{2}$	2.95	2.72	3.27	0.484	0	0.234	0.319	0.463	0.589	0.715	1
	1	2.73	2.70	2.76	0.453	0	0.249	0.337	0.482	0.608	0.730	1
	2	2.40	2.57	2.29	0.412	0	0.273	0.364	0.511	0.633	0.750	1
	3	2.17	2.30	2.03	0.390	0	0.289	0.383	0.530	0.650	0.762	1
	4	1.99	2.13	1.85	0.380	0	0.302	0.397	0.542	0.660	0.770	1
Ballistic nonlifting, $\frac{L}{D} = 0$	$-\varphi_1 > 5$	$3.32\left[30\ \sin(-\varphi)\right]^{-1/2}$	$3.30\left[30\ \sin(-\varphi)\right]^{-1/2}$	$3.22\left[30\ \sin(-\varphi)\right]^{-1/2}$	0.351	0	0.340	0.431	0.572	0.680	0.785	1

$^{*}\eta = \left(\frac{\lambda}{1 + \lambda}\right)^{1/2}$

Note: $\eta \approx \left(\frac{\lambda}{1 + \lambda}\right)^{\left(\bar{H}_{eff}\right)_{\lambda=0}}$

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It is seen from the table that the expected variation of mass loss with reentry time and maximum deceleration holds very well $\left(\text{within a few percent except for } L/D = 0, \ -\varphi_i < \frac{1}{2}^{\circ}\right)$ and that the lifting vehicle experiences greater mass loss than the nonlifting vehicle.

The effective heat capacity lies within definite limits, $0.35 \leq \left(\bar{H}_{eff}\right)_{\lambda=0} \leq 0.5$ the higher value being associated with lifting vehicles which operate relatively longer in the region of high stream enthalpy and therefore high gas layer shielding. It is seen that the variation of η with trajectory is small although its variation with λ is large.

Table I shows that $\eta \to 1$ as $\lambda \to \infty$ for all cases. Considering further the results as $\lambda \to \infty$, the effective heat capacity, H_{eff}, must tend to a limit L, the latent heat, or

$$\bar{H}_{eff} = \left(\bar{H}_{eff}\right)_{\lambda=0}(1 - \eta)^{-1} \to \lambda$$

This behavior may be verified for case (a) where

$$\left(\bar{H}_{eff}\right)_{\lambda=0}(1 - \eta)^{-1} = \frac{1}{2}\left[1 - \left(\frac{\lambda}{1 + \lambda}\right)^{1/2}\right]^{-1} \to \lambda$$

It is seen that $\bar{H}_{eff} \to \lambda$ because the exponent of $\frac{\lambda}{1 + \lambda}$ is equal to $\left(\bar{H}_{eff}\right)_{\lambda=0}\left(= \frac{1}{2}\right)$. This behavior suggests that η, which is equal to $\left(\frac{\lambda}{1 + \lambda}\right)^{1/2}$ for case (a) should be, more generally

$$\eta = \left(\frac{\lambda}{1 + \lambda}\right)^{\left(\bar{H}_{eff}\right)_{\lambda=0}} \tag{35}$$

and in fact a comparison shows that this result agrees with those in table I to within 1 percent.

In the foregoing analysis expressions for the total mass loss and the effective heat capacity of the shield have been obtained by assuming that (1) sublimation starts early during reentry and (2) the accumulation of heat is negligible compared with the disposal of heat; both assumptions lead to conservative values for the total mass loss (that is, values that are too large).

An analysis of the heat-conduction problem within the shield is desirable in order to justify these assumptions and also to estimate the amount of insulation required to keep the structure cool.

Accumulation of Heat Within the Shield

The effectiveness of an ablation shield in reducing heat transfer to the vehicle structure is measured finally in terms of the mass required to keep the structure below a given temperature. It has been shown in the previous section that the mass loss due to ablation is virtually independent of surface temperature when $T_e \gg T_s$. The mass requirements for insulation however depend critically on the ablation temperature and it is to be expected that the use of low-temperature ablation materials will reduce considerably the insulation problem with relatively little increase in the total mass loss.

In order to estimate this amount of insulation it is assumed that

$$\left(k_b \frac{\partial T}{\partial z}\right)_{z=0} = \frac{k_b(T_s - T_\infty)}{\theta} \tag{36}$$

(it may be shown that, for T_s monotonic increasing or constant

$$\left(k_b \frac{\partial T}{\partial z}\right)_{z=0} = A \frac{k_b(T_s - T_\infty)}{\theta}$$

where $A(t)$ lies in the range $\frac{2}{\pi} \leq A \leq 1$). With the substitution, equations (20) and (21) may be solved for the unknown quantities T_s and θ, when $\frac{dm}{dt} = 0$ (before ablation) or for m and θ when $T_s = T_a$ (during ablation).

Preablation heating.- Before appreciable ablation occurs (when the mass loss rate has negligible effect on the heat-transfer rate), equations (20), (21), and (36) give

$$q_0 = \rho_b c_b \frac{d}{dt}\left[(T_s - T_\infty)\theta\right] = \frac{k_b(T_s - T_\infty)}{\theta} \tag{37}$$

and elimination of θ gives

$$q_0 Q_0 = \rho_b c_b k_b (T_s - T_\infty)^2 \tag{38}$$

When sublimation begins $\bar{u} = \bar{u}_a$ and $T_s = T_a$ so that equation (38) written in dimensionless form, becomes

$$\bar{q}_0(\bar{u}_a)\bar{Q}_0(\bar{u}_a) = \frac{1.07}{k_I k_{II}} \frac{\rho_b c_b k_b}{\left(\frac{M}{C_D A R}\right)}(T_a - T_\infty)^2 \times 10^{-7} \tag{39}$$

(Strictly speaking, sublimation occurs at all values of the surface temperature and T_s is related to dm/dt through the phase relation which describes the equilibrium of the solid material with its vapor; in practice, however, dm/dt is negligible except when T_s lies within a limited range which includes the mean value T_a used here.)

Since $\frac{1.07}{k_I k_{II}} \approx 1$ it is seen from equation (39) that

$$\bar{q}_0 \bar{Q}_0 \approx \chi^2 \times 10^{-7} \tag{40}$$

where

$$\chi = \frac{(\rho_b c_b k_b)^{1/2}}{\left(\frac{M}{C_D A R}\right)^{1/2}}(T_a - T_\infty) \tag{41}$$

Equation (40) determines whether sublimation will occur during reentry; if

$$\left(\bar{q}_0\bar{Q}_0\right)_{max} < \chi^2 \times 10^{-7} \tag{42}$$

sublimation will not occur since equation (40) cannot be satisfied and the maximum value of T_s will remain below T_a throughout reentry. In terms of $\bar{u}$ and Z, the condition that sublimation will not occur is written

$$\chi^2 > 10^7 \left[\bar{u}^2(\bar{u}Z)^{1/2} \int_{\bar{u}}^{0.995} \bar{u}^2(\bar{u}Z)^{-1/2} d\bar{u}\right]_{max} \tag{43}$$

When χ is written

$$\chi = \frac{\rho_b c_b (T_a - T_\infty)}{\left(\frac{M}{C_D AR}\right)^{1/2}} \left(\frac{k_b}{\rho_b c_b}\right)^{1/2}$$

it is seen that sublimation cannot occur if

(1) the parameter $\left(\frac{M}{C_D AR}\right)^{1/2}$, which determines the level of the heating rate, is too small

(2) the thermal capacity $\rho_b c_b (T_a - T_\infty)$ is too large or

(3) the thermal diffusivity $\left(\frac{k_b}{\rho_b c_b}\right)^{1/2}$ is too large.

The materials under consideration in this paper have low ablation temperature and low thermal conductivity; more specifically the materials under serious consideration have properties with the following order of magnitude: $\rho_b \sim 0(10^2)$ lb/cu ft, $c_b \sim 0(1)$ Btu/lb °R, $k_b \sim 0\left(10^{-5}\right)$ Btu/ft sec °R, $T_a - T_\infty = 0\left(10^2, 10^3\right)$ °R; for vehicles considered here $\frac{M}{C_D AR} > 10^{-1}$. Thus $\chi < 10^2$ and sublimation will occur.

It is of interest to determine what fraction of the total flight time passes before sublimation takes place; the ratio t_a/t_f is found as follows:

Taking $t = 0$, when $\bar{u} = 0.995$, then since $\bar{q}_0$ is an increasing function of $\bar{u}$ during the early part of reentry

$$\bar{q}_0(\bar{u}_a) > \bar{q}_0(0.995) \quad \text{and} \quad \bar{Q}_0(\bar{u}_a) > \bar{t}_a\bar{q}_0(0.995)$$

therefore

$$\bar{q}_0(\bar{u}_a)\bar{Q}_0(\bar{u}_a) > \bar{q}_0^2(0.995)\bar{t}_a$$

also

$$\bar{t}_f = \int_{0.05}^{0.995} (\bar{u}Z)^{-1}d\bar{u}$$

so that

$$\frac{t_a}{t_f} = \frac{\bar{t}_a}{\bar{t}_f} < \frac{\bar{q}_0(u_a)\bar{Q}_0(u_a)}{\bar{q}_0^2(0.995)}\left[\int_{0.05}^{0.995}(\bar{u}Z)^{-1}d\bar{u}\right]^{-1}$$

$$\approx \left[Z(0.995)\int_{0.05}^{0.995}(\bar{u}Z)^{-1}d\bar{u}\right]^{-1} X^2 \times 10^{-7} \tag{44}$$

This ratio is evaluated later.

Total accumulation of heat.- An upper limit is found to the accumulation of heat during reentry by assuming that the surface of the shield is raised instantaneously to the temperature T_a at $t = 0$.

Equations (20), (21), and (36) are first combined to give

$$\rho_b\theta \frac{d}{dt}(\rho_b\theta) = \frac{\rho_b k_b}{c_b} - \rho_b\theta \frac{dm}{dt} \tag{45}$$

the last term is neglected, and integration then gives the following upper bound for θ

$$\rho_b\theta < \left(2\,\frac{\rho_b k_b}{c_b}\,t\right)^{1/2} \tag{46}$$

The accumulation of heat at time t is therefore

$$Q = \rho_b c_b(T_a - T_\infty)\theta < \left(2\rho_b c_b k_b\right)^{1/2}(T_a - T_\infty)t^{1/2} \tag{47}$$

and when $t = t_f$

$$Q_f < \left(2\rho_b c_b k_b\right)^{1/2}(T_a - T_\infty)t_f^{\,1/2}$$

This upper limit is independent of $\frac{M}{C_D AR}$ and for given material properties depends only on the total time of reentry. When Q_f is expressed as a fraction of $Q_{0,f}$ there is obtained

$$\frac{Q_f}{Q_{0,f}} < 0.46\,\frac{\bar{t}_f^{\,1/2}}{\bar{Q}_{0,f}}\,X \times 10^{-3} \tag{48}$$

where

$$\frac{t_f^{\,1/2}}{\bar{Q}_{0,f}} = \frac{\left[\int_{0.05}^{0.995}(\bar{u}Z)^{-1}d\bar{u}\right]^{1/2}}{\int_{0.05}^{0.995}\bar{u}^2(\bar{u}Z)^{-1/2}d\bar{u}} \tag{49}$$

and is evaluated by inserting the appropriate Z functions.

<u>Application to particular vehicles</u>.- The various functions of $\bar{u}$ and Z which appear in the previous section are evaluated by using the appropriate Z functions. The results may be summarized very concisely as follows. For all cases (a), (b), and (c), independent of L/D and φ_i

(1) Sublimation will not occur during reentry if

$$X^2 > 2 \times 10^6$$

(this condition does not hold necessarily if the shield is a composite slab of different materials; in such a case the factors $\rho_b c_b(\bar{T}_a - \bar{T}_\infty)$ and $\left(\frac{k_b}{\rho_b c_b}\right)^{1/2}$ which appear in the parameter χ would be replaced by the analogous composite quantities).

(2) The ratio $\frac{t_a}{t_f} = \frac{\text{Preablation heating period}}{\text{Total reentry time}}$ satisfies

$$\frac{t_a}{t_f} < 5 \times 10^{-6}\chi^2$$

(3) The ratio $\frac{Q_f}{Q_{0,f}} = \frac{\text{Heat accumulated by ablation shield}}{\text{Heat accumulated by heat sink}}$ satisfies

$$\frac{Q_f}{Q_{0,f}} < 1.5 \times 10^{-3}\chi$$

The assumptions, made in the analysis of sublimation, that sublimation begins soon after the initiation of reentry (when $\bar{u} = 0.995$) and that the accumulation of heat is small compared with the disposal of heat, are justified since

$$\frac{t_a}{t_f} < 0.05 \quad \text{and} \quad \frac{Q_0}{Q_{0,f}} < 0.15$$

for $\chi < 10^2$.

Insulation requirements.- The method of the preceding section gives an estimate only of the amount of heat accumulated by the solid shield at the completion of reentry. The temperature distribution through the shield is also of interest, however, and an approximate analysis is desirable, from which this distribution can be obtained. The nonlinear differential equations and boundary conditions for heat conduction in an ablating solid are developed, as in reference 10, but the mass loss rate

dm/dt is replaced by a constant mean value m/t_f. When this is done the equation becomes linear and has the solution at $t = t_f$.

$$\frac{T - T_\infty}{T_a - T_\infty} = e^{\epsilon_f \xi}\left[1 - \frac{1}{2}\,\mathrm{erfc}\left(\frac{\epsilon_f + \xi}{2}\right)\right] + \frac{1}{2}\,\mathrm{erfc}\left(\frac{\epsilon_f - \xi}{2}\right) \tag{50}$$

where

$$\xi = \left(\frac{\rho_b c_b}{k_b}\right)^{1/2} \frac{z}{t_f^{1/2}}$$

and

$$\epsilon_f = 0.17 k_{II} \frac{\left(\frac{M}{C_D A R}\right)^{1/2}}{\alpha \frac{\tilde{c}_p}{c_{p,2}}} (1 - \eta)\left(\frac{c_b}{\rho_b k_b}\right)^{1/2}$$

For engineering purposes equation (50) is easily approximated by

$$\frac{T - T_\infty}{T_a - T_\infty} = e^{z/\delta}, \quad \delta = \frac{2}{\pi^{1/2}}\left(\frac{k_b t_f}{\rho_b c_b}\right)^{1/2} \zeta(\epsilon_f) \tag{51}$$

where

$$\zeta(\epsilon_f) = \frac{1}{2} e^{-\epsilon_f^2/4} - \frac{\pi^{1/2}}{4}\epsilon_f\,\mathrm{erfc}\,\frac{\epsilon_f}{2} + \frac{\pi^{1/2}}{2\epsilon_f}\,\mathrm{erf}\,\frac{\epsilon_f}{2} \tag{52}$$

(δ is such that the expression for the heat content of the shield as given by eq. (50) agrees with that given by eqs. (51).)

The amount of insulation required to reduce the temperature from T_a to any value $T < T_a$ is then

$$m_{ins.} = \frac{2}{\pi^{1/2}}\left(\frac{\rho_b k_b}{c_b}\right)^{1/2} t_f^{1/2} \zeta(\epsilon_f) \ln \frac{T_a - T_\infty}{T - T_\infty} \ \text{lb/sq ft} \tag{53}$$

It may be verified that $\zeta(\epsilon_f) < 1$ and $\zeta(\epsilon_f) \to \frac{\pi^{1/2}}{2\epsilon_f}$ for large ϵ_f.

The importance of the shield material parameters $\frac{\rho_b k_b}{c_b}$ and T_a is evident from the foregoing expressions. It is seen also that the amount of insulation varies as $t_f^{1/2}$.

DISCUSSION

In the preceding analysis, the primary objective has been to obtain simple useful expressions to describe the sublimation of material from, and the accumulation of heat by a low-temperature, low conductivity shield suitable for manned reentry.

For the sake of simplicity several approximations have been made but they are of such a nature as to give conservative results, since upper limits have been obtained for the total mass loss due to sublimation and for the total heat accumulated during reentry.

It has been shown that the total mass required for sublimation depends primarily on the parameters

$$\frac{\left(\frac{M}{C_D A R}\right)^{1/2}}{\alpha \frac{\tilde{c}_p}{c_{p,2}}} = \frac{\text{Heating enthalpy coefficient}}{\text{Gas shielding enthalpy coefficient}}$$

and

$$\lambda = \frac{L + c_b(T_a - T_\infty)}{\frac{1}{2}\frac{u_c^2}{gJ}\alpha\frac{\tilde{c}_p}{c_{p,2}}}$$

$$= \frac{\text{Maximum internal shielding enthalpy}}{\text{Maximum external shielding enthalpy}}$$

the foregoing definition of λ gives the quasi-steady result whereas conservative results are obtained when $\lambda = \dfrac{L}{\frac{1}{2}\frac{u_c^2}{gJ}\,\alpha\,\frac{\tilde{c}_p}{c_{p,2}}}$. It is evident that the correct interpretation of $\alpha \dfrac{\tilde{c}_p}{c_{p,2}}$ is important in the use of these parameters; the quantity arises when the convective shielding in the boundary layer is considered and is correctly interpreted as

$$\frac{\tilde{c}_p}{c_{p,2}} = \frac{\text{Enthalpy of gases convected in boundary layer}}{\text{Enthalpy differences across boundary layer}}$$

The total mass loss during reentry can be written

$$m = 1.18k_I \frac{\left(\frac{M}{C_D A R}\right)^{1/2}}{\alpha \frac{\tilde{c}_p}{c_{p,2}}} \bar{m}_{\lambda=0}(1 - \eta) \quad \text{lb/sq ft}$$

where $\bar{m}_{\lambda=0}$ depends only on the vehicle trajectory; the exact values are given in table I although $\bar{m}_{\lambda=0}$ is given approximately by

$$\bar{m}_{\lambda=0} \approx \left(\frac{1}{30} \frac{\text{Max. horiz. decn., g's}}{2}\right)^{-1/2}$$

or

$$\bar{m}_{\lambda=0} \approx 0.84\left(\frac{t_f}{27}\right)^{1/2}$$

The quantity η, primarily a function of λ, represents the fractional reduction in mass loss due to latent heat.

The relation between total mass loss and maximum deceleration shows immediately the weight penalty incurred as the price of limiting the maximum deceleration to a low value and it is concluded that the use of a low-temperature ablation material is not appropriate to vehicles which have high L/D ratios (it is seen from table I, for example, that the value of $\bar{m}_{\lambda=0}$ for $L/D = 1/2$ is about twice that for $L/D = 0$).

The effective heat capacity of the ablation material is written

$$H_{eff} = 13{,}500\alpha \frac{\tilde{c}_p}{c_{p,2}}\left(\bar{H}_{eff}\right)_{\lambda=0}(1 - \eta)^{-1} \quad \text{Btu/lb}$$

where $\left(\bar{H}_{eff}\right)_{\lambda=0}$ and $\eta \approx \left(\frac{\lambda}{1 + \lambda}\right)^{\left(\bar{H}_{eff}\right)_{\lambda=0}}$ are dimensionless and do not vary appreciably with trajectory as seen in figures 4 and 5. Even when $\eta = 0$, (negligible latent heat) the effective heat capacity of the material is

$$13{,}500\alpha \frac{\tilde{c}_p}{c_{p,2}}\left(\bar{H}_{eff}\right)_{\lambda=0}$$

where

$$0.35 < \left(\bar{H}_{eff}\right)_{\lambda=0} \leq \frac{1}{2}$$

Again the importance of $\alpha \frac{\tilde{c}_p}{c_{p,2}}$ is seen; when $\alpha \frac{\tilde{c}_p}{c_{p,2}} = \frac{1}{2}$ for example the effective heat capacity, disregarding latent heat, is between 2,650 Btu/lb and 3,375 Btu/lb. When this range of effective heat capacities is compared with that for heat-sink metals of the order of 1,000 Btu/lb the reduction in shield weight is quickly realized. Moreover, since the foregoing comparison does not depend on the ablation temperature, the advantage is enhanced when low-temperature ablation materials are considered in view of the attendant reduction in insulation requirements.

For an ablation shield to perform successfully it must dispose of, rather than accumulate, heat energy; the preablation heating period should therefore be small compared with the total reentry time and the heat accumulated should be a small fraction of that which would be accumulated by a heat sink. Here, the deciding parameter is

$$\chi = \frac{\left(\rho_b c_b k_b\right)^{1/2}(T_a - T_\infty)}{\left(\frac{M}{C_D AR}\right)^{1/2}}$$

It has been shown that,

sublimation will not occur if $\chi^2 > 2 \times 10^6$

that, $$\frac{\text{Preablation heating period}}{\text{Total heating period}} < 5 \times 10^{-6}\chi^2$$

and

$$\frac{\text{Heat accumulated by ablation shield}}{\text{Heat accumulated by heat-sink shield}} < 1.5 \times 10^{-3}\chi$$

For the lifting vehicle $\left(\frac{L}{D} > \frac{1}{2} \text{ say}\right)$ the amount of mass loss will be large if ablation is allowed to take place over the major part of the reentry. Since such a vehicle would be cooled primarily by radiation, the use of a high temperature ablation material at the leading edge seems more appropriate. In such a design the ablation temperature should probably be near to the mean radiation temperature of the vehicle, and the parameter χ should be near the critical value 2×10^6 if ablation is to take place only near peak heating. The behavior of the ablation material during this long preablation period may be of concern however.

When the heat conduction problem is considered an upper limit to the accumulation of heat is found as

$$Q_f < \left(\rho_b c_b k_b t_f\right)^{1/2}(T_a - T_\infty) \quad \text{Btu/sq ft}$$

a result which is independent of the vehicle characteristics and heating experience except as they affect t_f.

Thus for a given vehicle the total shield weight required for sublimation and insulation varies approximately as the square root of the reentry time, or inversely as the square root of the maximum deceleration. For ballistic reentry, therefore ($-\varphi_i > 5^o$) and manned capsule reentry ($0 \leqq -\varphi_i < 5^o$) the ablation shield offers an efficient way to dispose of heat continuously during reentry. For the lifting vehicle a high temperature material which would allow radiation from the surface for the greater part of reentry appears to be more appropriate; ablation would then take place for a limited time near the maximum heating condition or in case of an emergency maneuver.

CONCLUDING REMARKS

An approximate analysis has been made of ablation shield requirements for reentry vehicles; the type of shield considered was one of low ablation temperature and low thermal conductivity which produced no liquid film during ablation.

It is shown that

1. The total mass required for sublimation depends primarily on parameters which depend on the ratios $\frac{\text{Heating enthalpy}}{\text{Gas shielding enthalpy}}$ and $\frac{\text{Shielding due to latent heat}}{\text{Gas layer shielding}}$.

2. For a given vehicle and shield the total mass loss varies as the square root of the total time for reentry or inversely as the square root of the maximum deceleration.

3. The accumulation of heat is a small percentage of that accumulated by a heat-sink shield, the percentage being determined by a single parameter which combines the effects of the heating level experienced during reentry, the thermal capacity of the remaining shield and the diffusivity of the material.

4. The amount of insulation material also varies as the square root of the time or inversely as the square root of the maximum deceleration.

From the foregoing dependence of sublimation and insulation requirements on deceleration and time of reentry it is concluded that the low temperature ablation shield should dispose of heat very efficiently for nonlifting vehicles, but the limited use of a high temperature ablation shield at the leading edges is more appropriate for lifting vehicles, where the primary means of cooling would be radiative.

APPENDIX

LIST OF SYMBOLS

A	reference area for drag and lift, sq ft
C_D	drag coefficient
c	specific heat, Btu/lb °R
g	gravitational acceleration, ft/sec^2
H_{eff}	effective heat capacity, Btu/lb
k	thermal conductivity, Btu/ft sec °R
k_I	ratio of local heat flux to that at stagnation point, $\frac{q_0}{q_{s_{p,0}}}$
k_{II}	average value of heat flux relative to stagnation point value, $\frac{1}{S}\int \frac{q_0}{q_{s_{p,0}}}\,dS$
L	latent heat of sublimation, Btu/lb
l	characteristic length of vehicle, ft
$\frac{L}{D}$	ratio, $\frac{\text{Lift force}}{\text{Drag force}}$
m	mass ablated, lb/sq ft
M	mass of vehicle, slugs
N_{Pr}	Prandtl number
N_{Sc}	Schmidt number
q	local convective heat transfer rate per unit area, Btu/ft^{2}sec
Q	total convective heat absorbed, Btu/sq ft
$\bar{q}$	dimensionless heat-transfer rate
$\bar{Q}$	dimensionless heat absorbed per unit area
r	distance from center of earth to orbit

R	radius of curvature of nose
Re	Reynolds number, $\frac{\rho_\infty V l}{\mu_\infty}$
S	surface area wetted by boundary layer, ft^2
t	time, sec
T	temperature, ^oR
u	circumferential velocity component, ft/sec
u_c	circular orbital velocity, $(gr)^{1/2} = 26{,}000$ ft/sec
$\bar{u}$	ratio, $\frac{u}{u_c}$
v	temperature ratio, $\frac{T - T_\infty}{T_a - T_\infty}$
Y	altitude, ft
V	total velocity, ft/sec
W	weight of vehicle at earth's surface, lb
Z	dimensionless function of $\bar{u}$ determined by equation (4)
y	outward normal distance from initial position of ablation surface, ft
z	outward normal distance from ablation surface, ft
a	fractional temperature rise of gaseous material
ϵ	dimensionless ablation rate
β	atmospheric density decay parameter, ft^{-1}
θ	integral thickness of heated layer in solid shield, ft
μ	coefficient of dynamic viscosity, slug/ft sec
λ	latent heat parameter, equation (33) or (34)
ρ	density, slug/cu ft
η	fractional decrease in mass loss due to latent heat-dimensionless distance

τ dimensionless time

φ flight path angle relative to local horizontal direction; negative for descent

χ conduction parameter

Subscripts:

0 no sublimation

∞ free stream, also conditions before reentry

s surface condition

sp stagnation point

i initial condition

1 gas produced by sublimation

2 air behind shock wave

a sublimation condition

b solid shield condition

e external to boundary layer at stagnation point

f final conditions

Superscripts:

' differentiation with respect to $\bar{u}$

$-$ dimensionless quantity

$\sim$ mean value

$*$ mean value for exponential approximation to density-altitude relationship

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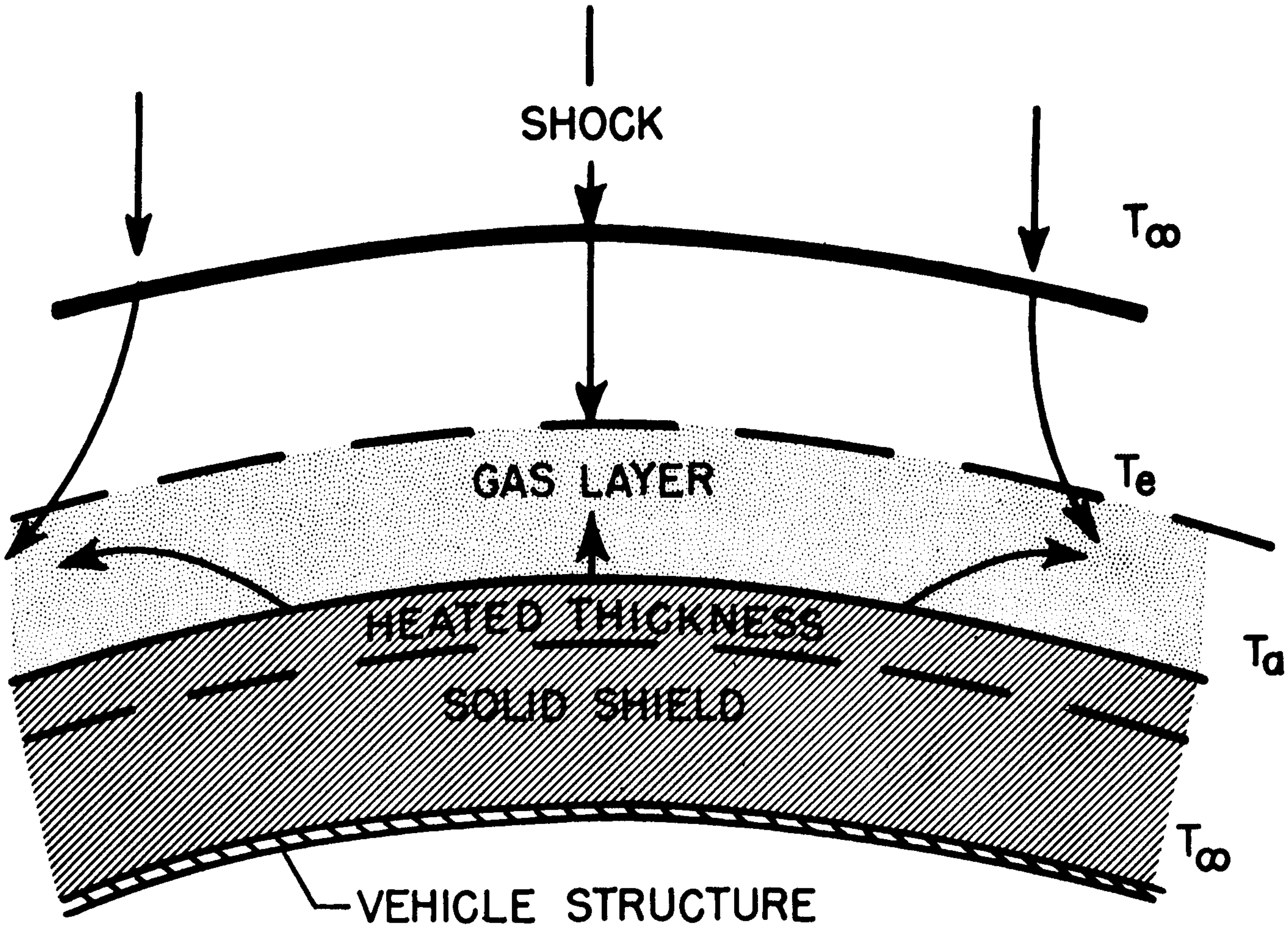

Figure 1.- The ablation shield.